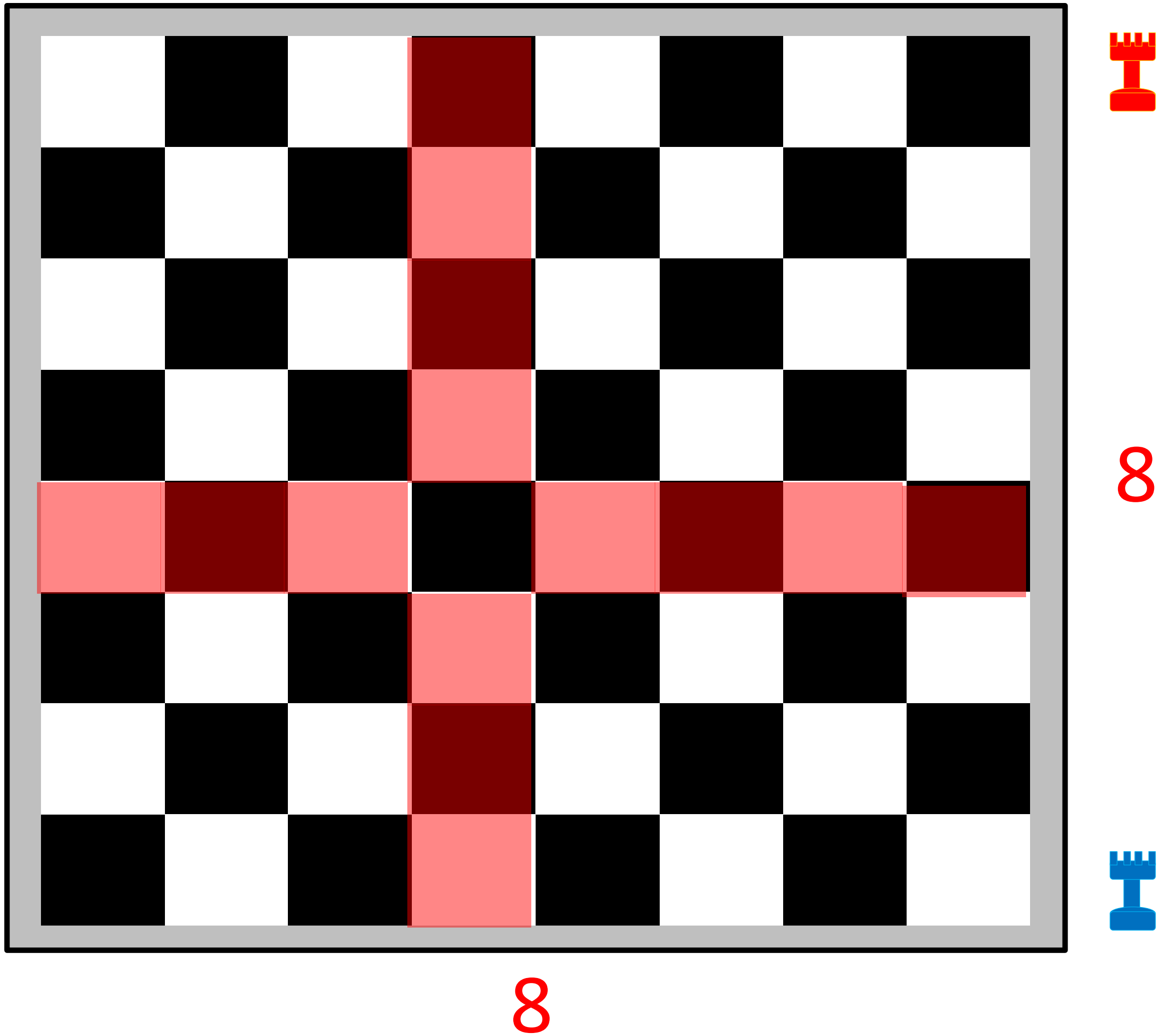
Generalisation of the Probability and Chess Question (Step-by-Step)

Omkar Prakash Sambare
Ramanujan Academy
Nasik, Maharashtra
omkarpsambare@gmail.com

Q. There is a normal chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?



Set S be the all possible places for second rooks

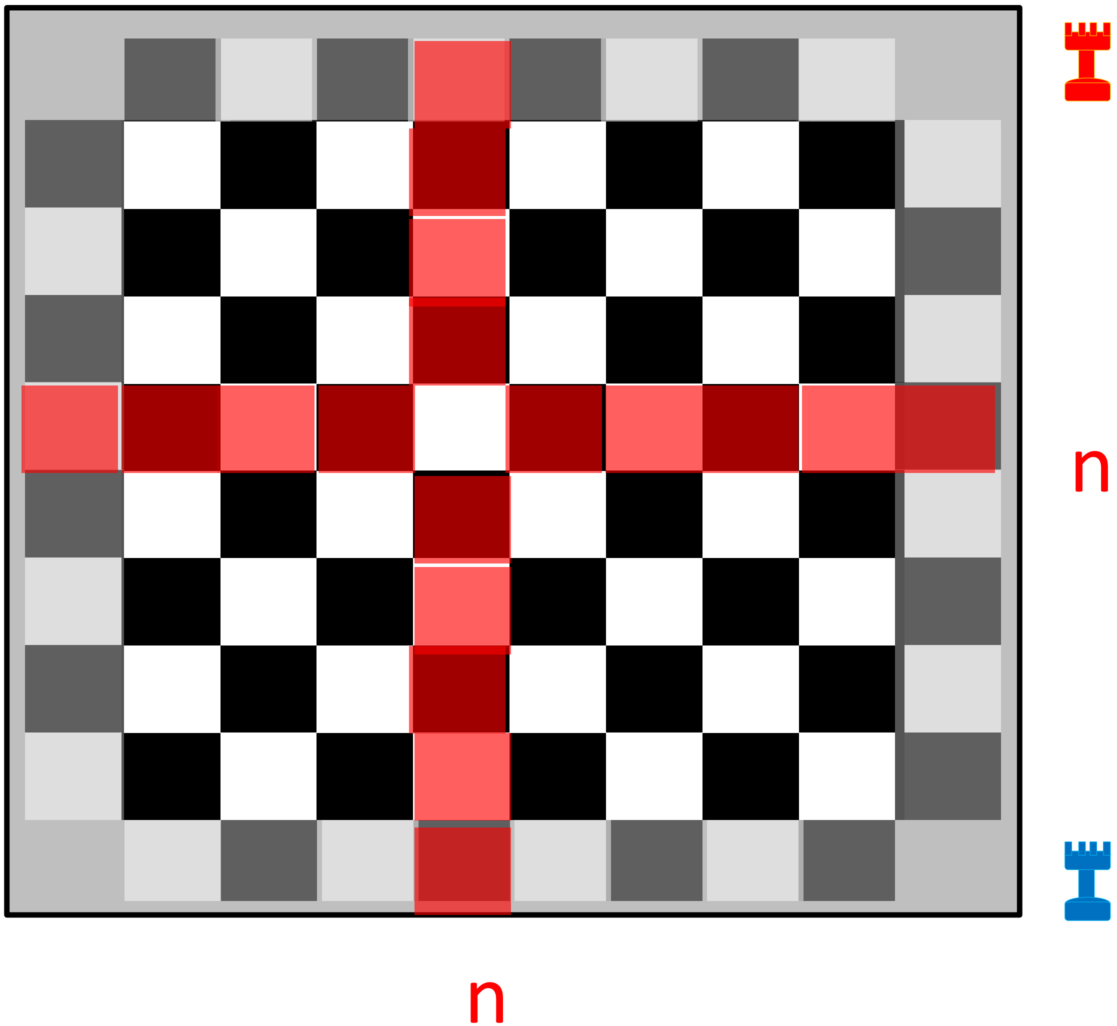
$$n(S) = 8 \times 8 - 1 = 63$$

Set R be the defined places for second rooks

$$n(R) = (8-1) + (8-1)$$
$$n(R) = 14$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{14}{63} = \frac{2}{9}}$$

Q. There is $(\boldsymbol{n} \times \boldsymbol{n})$ chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?



Set S be the all possible places for second rooks

$$n(S) = n \times n - 1 = n^2 - 1$$

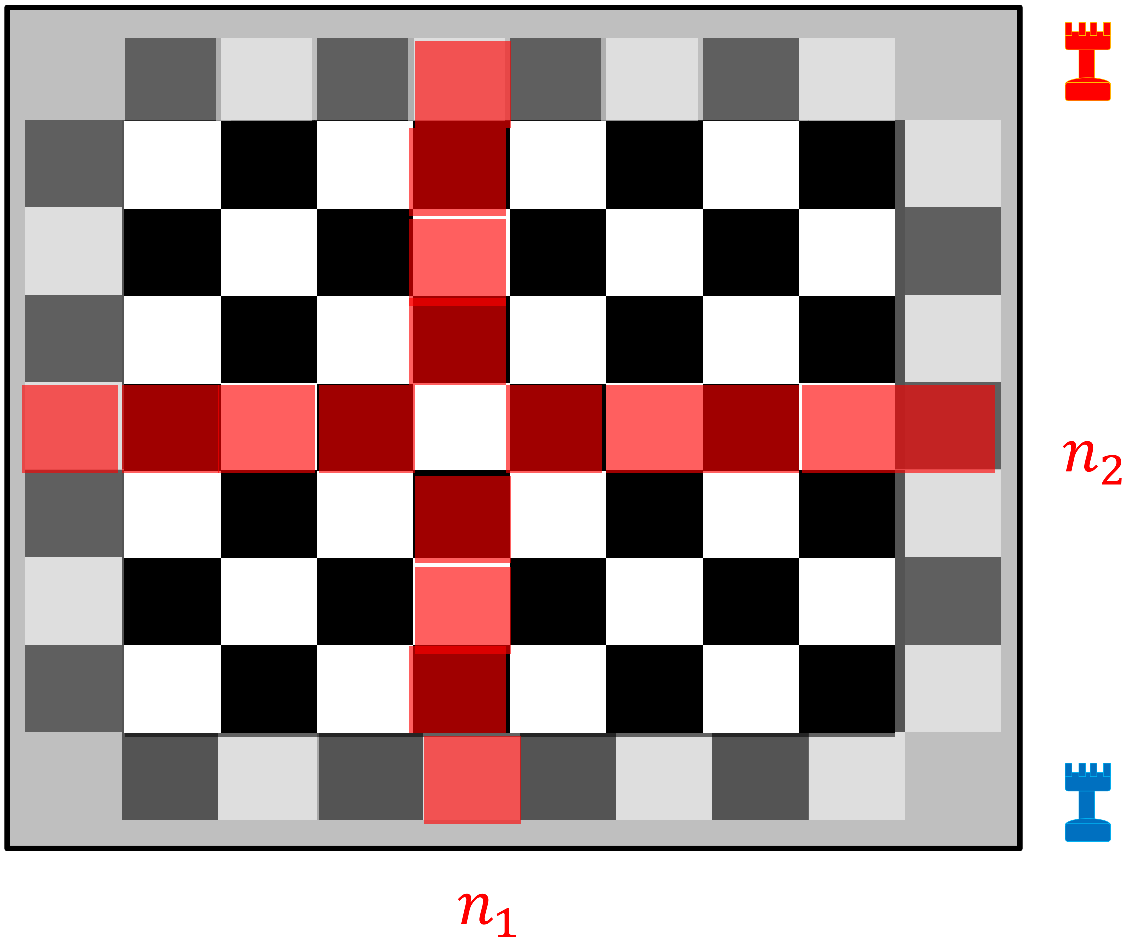
Set R be the defined places for second rooks

$$n(R) = (n-1) + (n-1)$$
$$n(R) = 2(n-1)$$

$$P(R) = \frac{n(R)}{n(S)} = \frac{2(n-1)}{n^2-1}$$

$$\boldsymbol{P(R) = \frac{2}{n+1}}$$

Q. There is $(\boldsymbol{n_1} \times \boldsymbol{n_2})$ chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?



Set S be the all possible places for second rooks

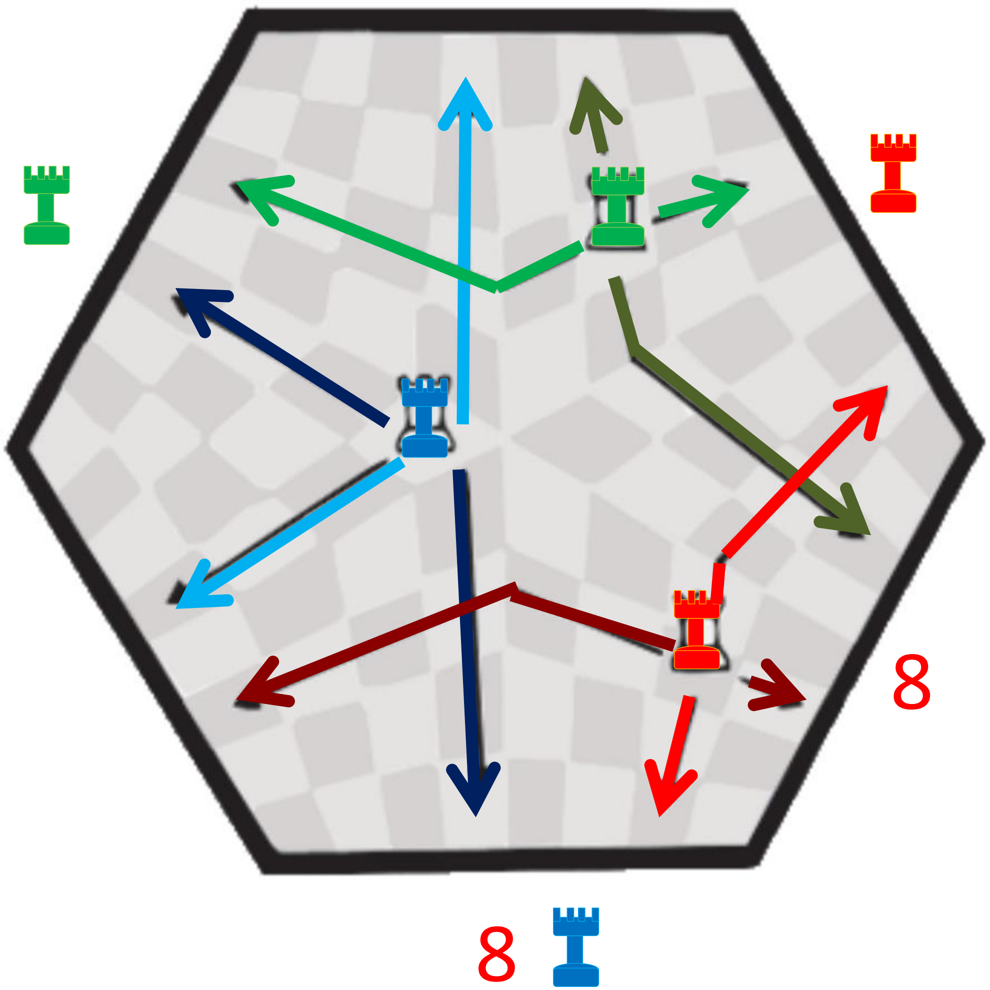
$$n(S) = n_1.n_2 - 1$$

Set R be the defined places for second rooks

$$n(R) = (n_1 - 1) + (n_2 - 1)$$
$$n(R) = n_1 + n_2 - 2$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{n_1 + n_2 - 2}{n_1.n_2 - 1}}$$

Q. There is a (8×8) **3 players** chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?



Total squares

$$= 3\left(\frac{8^2}{2}\right) = 96$$

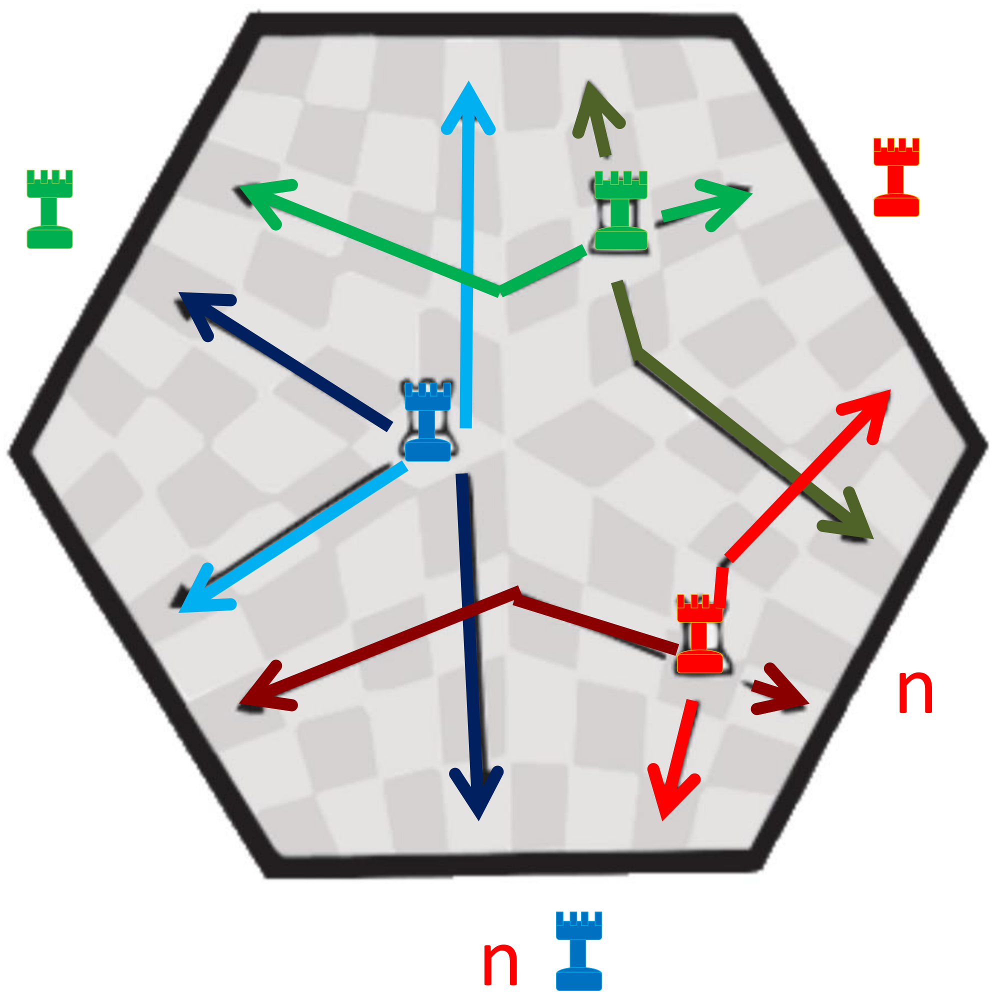
$$n(S) = 96 - 1 = 95$$

Number of attacking squares of 1st rook

$$n(R) = 2 \times 7 = 14$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{14}{95}}$$

Q. There is a $(n \times n)$ **3 players** chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?



Total squares

$$= 3\left(\frac{n^2}{2}\right) = \frac{3n^2}{2}$$

$$n(S) = \frac{3n^2}{2} - 1 = \frac{3n^2 - 2}{2}$$

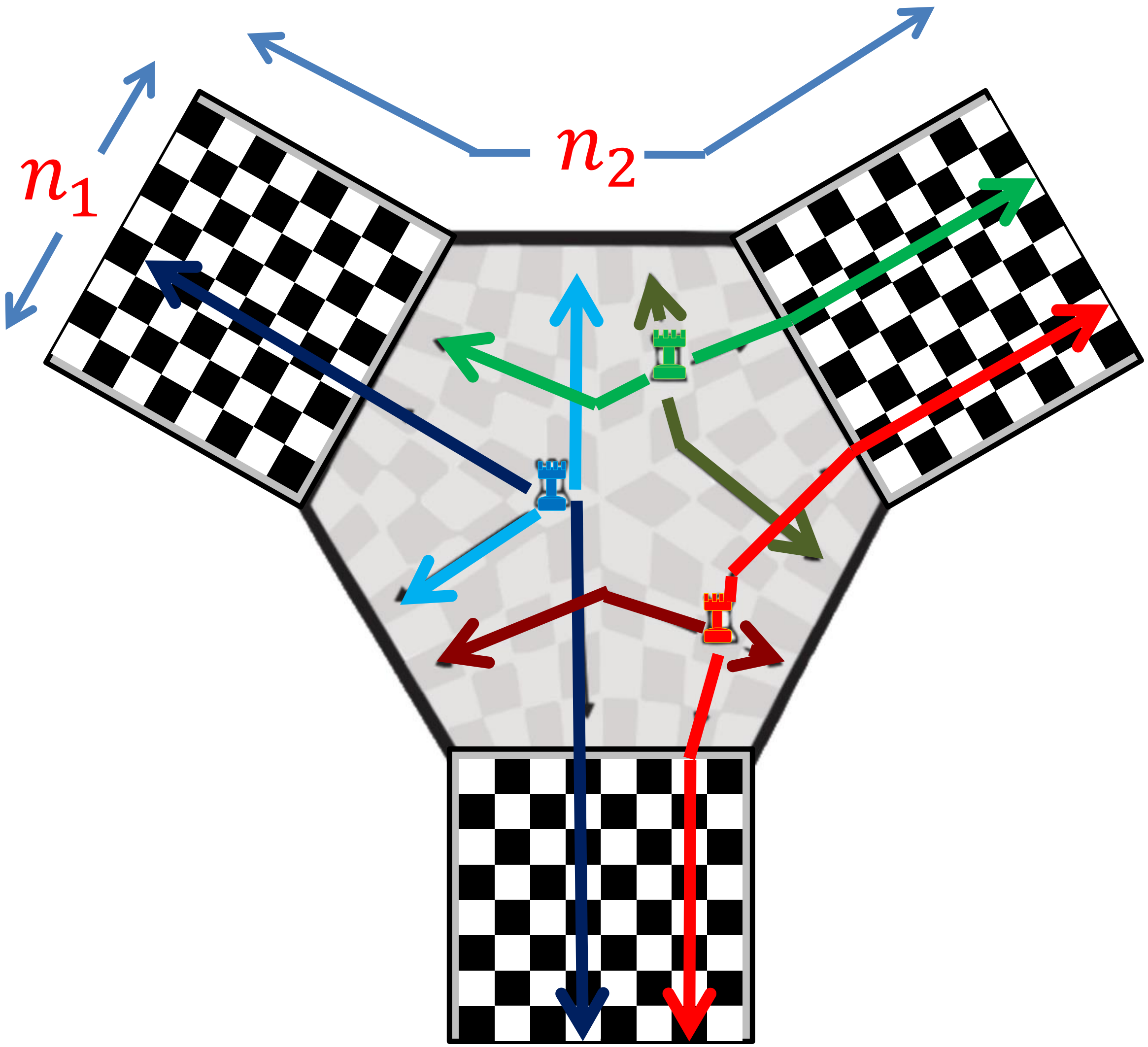
Number of attacking squares of 1st rook

$$n(R) = 2 \times (n-1)$$

$$P(R) = \frac{n(R)}{n(S)} = \frac{2 \times (n-1)}{\left(\frac{3n^2-2}{2}\right)}$$

$$\boldsymbol{P(R) = \frac{4 \times (n-1)}{3n^2 - 2}}$$

Q. There is a $(n_1 \times n_2)$ **3 players** chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?



Total squares

$$= 3\left(\frac{n_1.n_2}{2}\right) = \frac{3n_1.n_2}{2}$$

$$n(S) = \frac{3n_1.n_2}{2} - 1 = \frac{3n_1.n_2 - 2}{2}$$

Number of attacking squares of 1st rook

$$n(R) = (n_1 - 1) + (n_2 - 1)$$

$$n(R) = n_1 + n_2 - 2$$

$$P(R) = \frac{n(R)}{n(S)} = \frac{n_1 + n_2 - 2}{\left(\frac{3n_1.n_2 - 2}{2}\right)}$$

$$\boldsymbol{P(R) = \frac{2 \times (n_1 + n_2 - 2)}{3n_1.n_2 - 2}}$$

Different number of players chess board

(Using “CHNESS” android app)

Q. There is a **(8 × 8)** **z players** chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

Total squares $= z\left(\frac{8^2}{2}\right) = 32Z$ $\qquad n(S) = 32z - 1$

Number of attacking squares of 1st rook $\qquad n(R) = 2 \times 7 = 14$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{14}{32z-1}}$$

Q. There is a **$(n \times n)$ z players** chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

Total squares $= z\left(\frac{n^2}{2}\right) = \frac{zn^2}{2}$ $\qquad n(S) = \frac{zn^2}{2} - 1 = \frac{zn^2-2}{2}$

Number of attacking squares of 1st rook $\qquad n(R) = 2 \times (n-1)$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{2 \times (n-1)}{\left(\frac{zn^2-2}{2}\right)} = \frac{4 \times (n-1)}{zn^2-2}}$$

Q. There is a $(\boldsymbol{n_1 \times n_2})$ **z players** chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

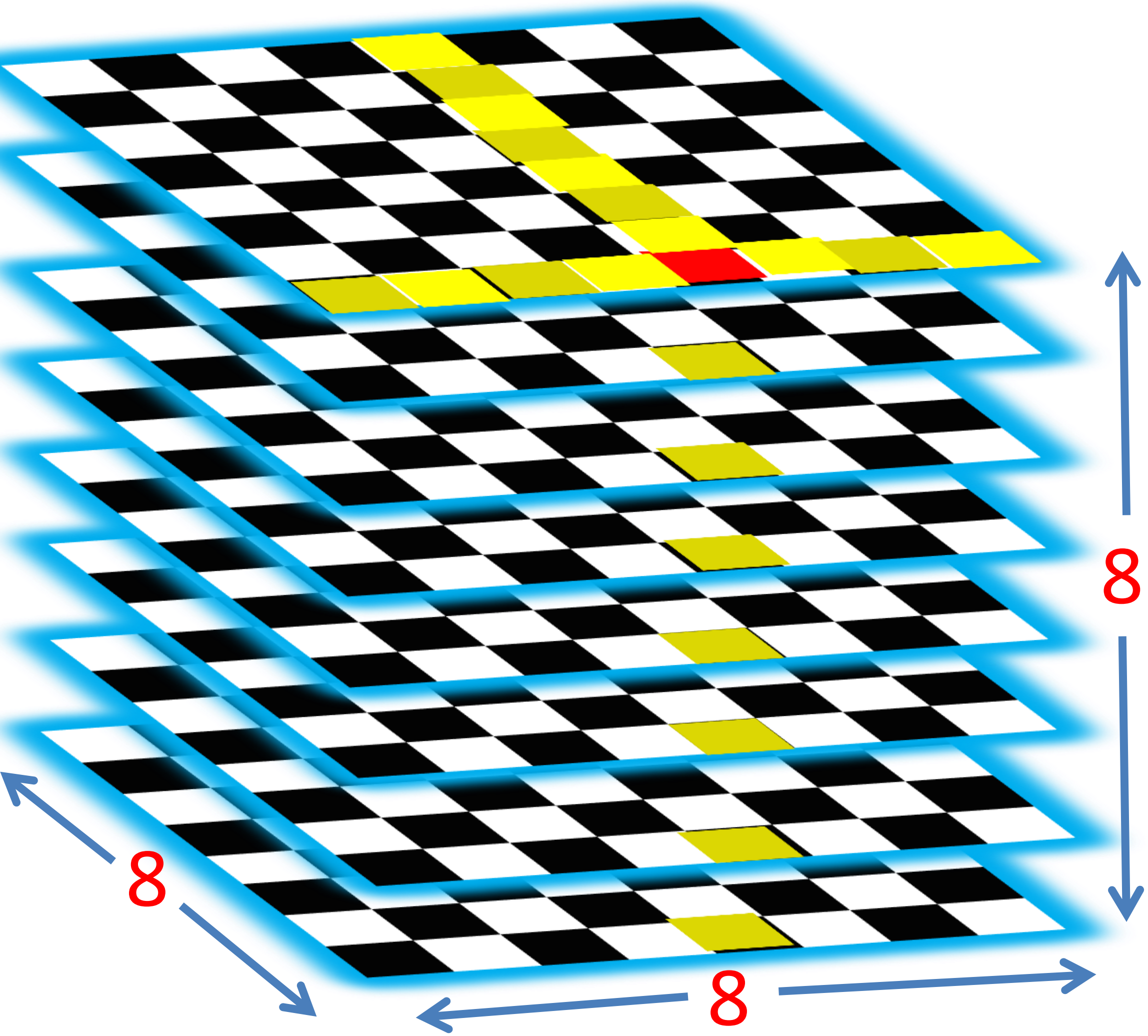
Total squares $= z\left(\frac{\boldsymbol{n_1 \times n_2}}{2}\right) = \frac{zn_1n_2}{2}$ $\quad n(S) = \frac{zn_1n_2}{2} - 1 = \frac{zn_1n_2 - 2}{2}$

Number of attacking squares of 1st rook $\quad n(R) = (n_1 - 1) + (n_2 - 1) = n_1 + n_2 - 2$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{n_1 + n_2 - 2}{\left(\frac{zn_1n_2 - 2}{2}\right)} = \frac{2 \times (n_1 + n_2 - 2)}{zn_1n_2 - 2}}$$

Again come back to original question and make some changes

Q. There is a 3 Dimensional $(8 \times 8 \times 8)$ chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?



Set S be the all possible places for second rooks

$$n(S) = 8 \times 8 \times 8 - 1 = 511$$

Set R be the defined places for second rooks

$$n(R) = (8-1) + (8-1) + (8-1)$$
$$n(R) = 21$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{21}{511} = \frac{3}{73}}$$

Q. There is a 3 Dimensional $(n \times n \times n)$ chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

3 Dimensional $(8 \times 8 \times 8)$

Set S be the all possible places for second rooks

$$n(S) = 8 \times 8 \times 8 - 1 = 511$$

Set R be the defined places for second rooks

$$n(R) = (8-1) + (8-1) + (8-1)$$
$$n(R) = 21$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{21}{511} = \frac{3}{73}}$$

3 Dimensional $(n \times n \times n)$

Set S be the all possible places for second rooks

$$n(S) = n \times n \times n - 1 = n^3 - 1$$

Set R be the defined places for second rooks

$$n(R) = (n-1) + (n-1) + (n-1)$$
$$n(R) = 3 \times (n-1)$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{3 \times (n-1)}{n^3 - 1} = \frac{3}{(n^2 + n + 1)}}$$

Q. There is a 3 Dimensional $(n_1 \times n_2 \times n_3)$ chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

3 Dimensional $(n \times n \times n)$

Set S be the all possible places for second rooks

$$n(S) = n \times n \times n - 1 = n^3 - 1$$

Set R be the defined places for second rooks

$$n(R) = (n-1) + (n-1) + (n-1)$$
$$n(R) = 3 \times (n-1)$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{3 \times (n-1)}{n^3 - 1} = \frac{3}{(n^2 + n + 1)}}$$

3 Dimensional $(n_1 \times n_2 \times n_3)$

Set S be the all possible places for second rooks

$$n(S) = n_1.n_2.n_3 - 1$$

Set R be the defined places for second rooks

$$n(R) = (n_1 - 1) + (n_2 - 1) + (n_3 - 1)$$
$$n(R) = n_1 + n_2 + n_3 - 3$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{n_1 + n_2 + n_3 - 3}{n_1.n_2.n_3 - 1}}$$

Q. There is a 3 Dimensional $(8 \times 8 \times 8)$ **3 players** chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

3 Dimensional $(8 \times 8 \times 8)$ 2 players

$$n(S) = 8^3 - 1 = 511$$

$$n(R) = (8-1) + (8-1) + (8-1) = 21$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{21}{511} = \frac{3}{73}}$$

3 Dimensional $(8 \times 8 \times 8)$ 3 players

$$n(S) = 3 \times \left(\frac{8^3}{2}\right) - 1 = 767$$

$$n(R) = (8-1) + (8-1) + (8-1) = 21$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{21}{767}}$$

Q. There is a 3 Dimensional $(8 \times 8 \times 8)$ **z players** chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

$$n(S) = \boldsymbol{z} \times \left(\frac{8^3}{2}\right) - 1 = 256z - 1$$

$$n(R) = (8-1) + (8-1) + (8-1) = 21$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{21}{256z - 1}}$$

Q. There is a 3 Dimensional $(\boldsymbol{n} \times \boldsymbol{n} \times \boldsymbol{n})$ 3 players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

3 Dimensional $(n \times n \times n)$ 2 players

$$n(S) = n^3 - 1$$

$$n(R) = (n-1) + (n-1) + (n-1)$$

$$n(R) = 3 \times (n-1)$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{3 \times (n-1)}{n^3 - 1} = \frac{3}{(n^2 + n + 1)}}$$

3 Dimensional $(n \times n \times n)$ 3 players

$$n(S) = 3 \times \left(\frac{n^3}{2}\right) - 1 = \frac{3n^3 - 2}{2}$$

$$n(R) = (n-1) + (n-1) + (n-1)$$

$$n(R) = 3 \times (n-1)$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{3 \times (n-1)}{\left(\frac{3n^3 - 2}{2}\right)} = \frac{6 \times (n-1)}{3n^3 - 2}}$$

Q. There is a 3 Dimensional $(n \times n \times n)$ **z** players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

$$n(S) = z \times \left(\frac{n^3}{2}\right) - 1 = \frac{zn^3 - 2}{2}$$

$$n(R) = 3 \times (n-1)$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{3 \times (n-1)}{\left(\frac{zn^3 - 2}{2}\right)} = \frac{6 \times (n-1)}{zn^3 - 2}}$$

Q. There is a 3 Dimensional $(\boldsymbol{n_1 \times n_2 \times n_3})$ 3 players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

3 Dimensional $(n_1 \times n_2 \times n_3)$ 2 players

$$n(S) = n_1.n_2.n_3 - 1$$

$$n(R) = (n_1 - 1) + (n_2 - 1) + (n_3 - 1)$$
$$n(R) = n_1 + n_2 + n_3 - 3$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{n_1 + n_2 + n_3 - 3}{n_1.n_2.n_3 - 1}}$$

3 Dimensional $(n_1 \times n_2 \times n_3)$ 3 players

$$n(S) = 3 \times \left(\frac{n_1.n_2.n_3}{2}\right) - 1 = \frac{3n_1.n_2.n_3 - 2}{2}$$

$$n(R) = (n_1 - 1) + (n_2 - 1) + (n_3 - 1)$$
$$n(R) = n_1 + n_2 + n_3 - 3$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{n_1 + n_2 + n_3 - 3}{\left(\frac{3.n_1.n_2.n_3 - 2}{2}\right)}}$$
$$\boldsymbol{= \frac{2(n_1 + n_2 + n_3 - 3)}{3.n_1.n_2.n_3 - 2}}$$

Q. There is a 3 Dimensional $(n_1 \times n_2 \times n_3)$ **z** players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

$$n(S) = \boldsymbol{z} \times \left(\frac{n_1.n_2.n_3}{2}\right) - 1 = \frac{z.n_1.n_2.n_3 - 2}{2}$$

$$n(R) = (n_1 - 1) + (n_2 - 1) + (n_3 - 1)$$
$$n(R) = n_1 + n_2 + n_3 - 3$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{n_1 + n_2 + n_3 - 3}{\left(\frac{z.n_1.n_2.n_3 - 2}{2}\right)}}$$
$$\boldsymbol{= \frac{2(n_1 + n_2 + n_3 - 3)}{z.n_1.n_2.n_3 - 2}}$$

Q. There is $\mathbf{Y}$ Dimensional $(8 \times 8 \times \cdots \times 8)$ 2 players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

$$n(S) = 8 \times 8 \times 8 \times \cdots \times 8 - 1 = 8^{\boldsymbol{Y}} - 1$$

$$n(R) = (8-1) + (8-1) + (8-1) + \cdots (Y\ times) = \mathrm{Y}(8-1) = 7\mathrm{Y}$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{7Y}{8^{Y} - 1}}$$

Q. There is $\mathbf{Y}$ Dimensional $(n \times n \times \cdots \times n)$ 2 players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

$$n(S) = n \times n \times n \times \cdots \times n - 1 = \boldsymbol{n}^{Y} - 1$$

$$n(R) = (n-1) + (n-1) + (n-1) + \cdots (Y\ times) = \mathrm{Y}(\mathrm{n}-1)$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{Y(n-1)}{n^{Y} - 1}}$$

Q. There is $\mathbf{Y}$ Dimensional $(n \times n \times \cdots \times n)$ 2 players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

$$n(S) = n \times n \times n \times \cdots \times n - 1 = \boldsymbol{n}^{Y} - 1$$

$$n(R) = (n-1) + (n-1) + (n-1) + \cdots (Y\ times) = \mathrm{Y}(\mathrm{n}-1)$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{Y(n-1)}{n^{Y}-1}}$$

Q. There is $\mathbf{Y}$ Dimensional $(n_1 \times n_2 \times n_3 \times \cdots \times n_Y)$ 2 players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

$$n(S) = n_1 \times n_2 \times n_3 \times \cdots \times n_Y - 1$$

$$n(R) = (n_1 - 1) + (n_2 - 1) + (n_3 - 1) + \cdots + (n_Y - 1) = n_1 + n_2 + n_3 + \cdots + n_Y - Y$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{n_1 + n_2 + n_3 + \cdots + n_Y - Y}{n_1 \times n_2 \times n_3 \times \cdots \times n_Y - 1}}$$

Q. There is Y Dimensional $(8 \times 8 \times \cdots \times 8)$ **z** players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

$$n(S) = \frac{\boldsymbol{z} \times (8 \times 8 \times 8 \times \cdots \times 8)}{2} - 1 = \frac{z8^Y}{2} - 1 = \frac{z8^Y - 2}{2}$$

$$n(R) = (8-1) + (8-1) + (8-1) + \cdots (Y\ times) = \mathrm{Y}(8-1) = 7\mathrm{Y}$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{7Y}{\left(\frac{z8^Y - 2}{2}\right)} = \frac{14Y}{z8^Y - 2}}$$

Q. There is Y Dimensional $(n \times n \times \cdots \times n)$ z players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

$$n(S) = \frac{z \times (n \times n \times n \times \cdots \times n)}{2} - 1 = \frac{zn^Y}{2} - 1 = \frac{zn^Y - 2}{2}$$

$$n(R) = (n-1) + (n-1) + (n-1) + \cdots (Y\ times) = \mathrm{Y(n-1)} = \mathrm{Y(n-1)}$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{Y(n-1)}{\left(\frac{zn^Y - 2}{2}\right)} = \frac{2Y(n-1)}{zn^Y - 2}}$$

Q. There is Y Dimensional $(n \times n \times \cdots \times n)$ z players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

$$n(S) = \frac{z \times (n \times n \times n \times \cdots \times n)}{2} - 1 = \frac{zn^Y}{2} - 1 = \frac{zn^Y - 2}{2}$$

$$n(R) = (n-1) + (n-1) + (n-1) + \cdots (Y\ times) = \mathrm{Y(n-1)} = \mathrm{Y(n-1)}$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{Y(n-1)}{\left(\frac{zn^Y - 2}{2}\right)} = \frac{2Y(n-1)}{zn^Y - 2}}$$

Q. There is Y Dimensional $(n_1 \times n_2 \times \cdots \times n_Y)$ z players chess board and two rooks. If we put that 2 rooks on squares chessboard one-by-one randomly, then what is the probability that two rooks are attacking on each other?

$$n(S) = \frac{z \times (n_1 \times n_2 \times \cdots \times n_Y)}{2} - 1 = \frac{z \times n_1 \times n_2 \times \cdots \times n_Y}{2} - 1 = \frac{zn_1n_2 \ldots n_Y - 2}{2}$$

$$n(R) = (n_1 - 1) + (n_2 - 1) + (n_3 - 1) + \cdots + (n_Y - 1) = n_1 + n_2 + n_3 + \cdots + n_Y - Y$$

$$\boldsymbol{P(R) = \frac{n(R)}{n(S)} = \frac{n_1 + n_2 + n_3 + \cdots + n_Y - Y}{\frac{zn_1n_2 \ldots n_Y - 2}{2}} = \frac{2(n_1 + n_2 + n_3 + \cdots + n_Y - Y)}{zn_1n_2 \ldots n_Y - 2}}$$